

C) Formule goniometriche.

1) Formule di addizione.

$$\begin{cases} \sin(\alpha + \beta) = \sin\alpha \cos\beta + \sin\beta \cos\alpha \\ \cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta \\ \operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \operatorname{tg}\beta} \end{cases}$$

2) Formule di sottrazione⁽¹⁾.

$$\begin{cases} \sin(\alpha - \beta) = \sin\alpha \cos\beta - \sin\beta \cos\alpha \\ \cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta \\ \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha \operatorname{tg}\beta} \end{cases}$$

3) Formule di duplicazione.

$$\begin{cases} \sin 2\alpha = 2 \sin\alpha \cos\alpha \\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2 \sin^2\alpha = 2 \cos^2\alpha - 1 \\ \operatorname{tg} 2\alpha = \frac{2 \operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha} \end{cases}$$

4) Formule di triplicazione.

$$\begin{cases} \sin 3\alpha = 3 \sin\alpha - 4 \sin^3\alpha \\ \cos 3\alpha = 4 \cos^3\alpha - 3 \cos\alpha \\ \operatorname{tg} 3\alpha = \frac{3 \operatorname{tg}\alpha - \operatorname{tg}^3\alpha}{1 - 3 \operatorname{tg}^2\alpha} \end{cases}$$

5) Formule di bisezione.

$$\begin{cases} \sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos\alpha}{2}}, \quad \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos\alpha}{2}} \\ \operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}, \quad = \frac{\sin\alpha}{1 + \cos\alpha} = \frac{1 - \cos\alpha}{\sin\alpha} \end{cases}$$

6) Formule di prostaferesi.

$$\begin{cases} \sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2} \\ \sin p - \sin q = 2 \cos \frac{p+q}{2} \sin \frac{p-q}{2} \\ \cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2} \\ \cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2} \end{cases}$$

7) Formule di Werner.

$$\begin{cases} \sin\alpha \sin\beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos\alpha \cos\beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] \\ \sin\alpha \cos\beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)] \end{cases}$$