Formule e tavole per l'esame di Statistica II

- Se $(Y_1, \ldots, Y_n)$ sono variabili casuali indipendenti ed identicamente distribuite come una $N(\mu, \sigma^2)$ allora

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

  e quindi

$$\frac{\sqrt{n}(\overline{Y} - \mu)}{\sigma} \sim N(0,1)$$

- Se $(Y_1, \ldots, Y_n)$ sono variabili casuali indipendenti ed identicamente distribuite come una $N(\mu, \sigma^2)$ e S^2 è lo stimatore corretto di σ^2, allora

$$\frac{\sqrt{n}(\overline{Y} - \mu)}{S} \sim t_{n-1} \qquad (t \text{ di Student con } n-1 \text{ gradi di libertà.})$$

- Se $Y \sim \mathrm{Bi}(n, \vartheta)$, per n non troppo piccolo, la distribuzione di

$$\frac{\hat{\vartheta} - \vartheta}{\sqrt{\vartheta(1-\vartheta)/n}},$$

  dove $\hat{\vartheta} = Y/n$, è approssimabile con quella di una normale standard.

- Se $(y_{11}, \ldots, y_{1n_1})$ è un campione casuale semplice estratto da una variabile casuale $Y_1 \sim N(\mu_1, \sigma^2)$ e $(y_{21}, \ldots, y_{2n_2})$ è un campione casuale semplice, indipendente dal precedente, estratto da una variabile casuale $Y_2 \sim N(\mu_2, \sigma^2)$, allora

$$\frac{\overline{Y}_1 - \overline{Y}_2 - (\mu_1 - \mu_2)}{S\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

  dove $\overline{Y}_1$ e $\overline{Y}_2$ sone le medie dei due gruppi mentre

$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum_{i=1}^{n_1}(Y_{1i} - \overline{Y}_1)^2 + \sum_{i=1}^{n_2}(Y_{2i} - \overline{Y}_2)^2\right]$$

- Se $Y_1 \sim \mathrm{Bi}(n_1, \vartheta_1)$ e $Y_2 \sim \mathrm{Bi}(n_2, \vartheta_2)$ sono indipendenti, per n_1 e n_2 e non troppo piccoli, la distribuzione di

$$\frac{\hat{\vartheta}_1 - \hat{\vartheta}_2 - (\vartheta_1 - \vartheta_2)}{\sqrt{\vartheta_1(1-\vartheta_1)/n_1 + \vartheta_2(1-\vartheta_2)/n_2}},$$

  dove $\hat{\vartheta}_1 = Y_1/n_1$ e $\hat{\vartheta}_2 = Y_2/n_2$, è approssimabile con quella di una normale standard.

- Se la più piccola delle frequenze attese della tabella con r righe e c colonne non è troppo piccola la distribuzione dell'X^2 di Pearson

$$X^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(f_{ij} - \hat{f}_{ij})^2}{\hat{f}_{ij}},$$

  dove $\hat{f}_{ij} = f_{i+}f_{+j}/n$, può essere approssimata con quella di una variabile casuale χ^2 con $(r-1)\times(c-1)$ gradi di libertà.

- In un modello di regressione lineare semplice $Y_i = \alpha + \beta x_i + \varepsilon_i$, le stime con il metodo dei minimi quadrati dei parametri α e β e la stima di σ^2 sono

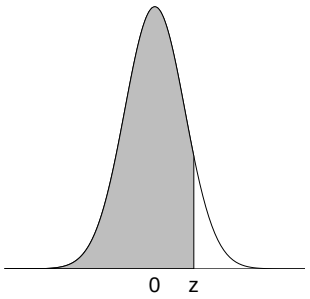
$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}, \qquad \hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}, \qquad \hat{\sigma}^2 = \frac{\sum_{i=1}^{n} y_i^2 - n\overline{y}^2 - \hat{\beta}^2\left(\sum_{i=1}^{n} x_i^2 - n\overline{x}^2\right)}{n-2}$$

  Inoltre, se le v.c. Y_i hanno distribuzione normale, si ha

$$\begin{aligned}\hat{\alpha} &\sim \mathcal{N}\left(\alpha, \left(\frac{1}{n} + \frac{\overline{x}^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}\right)\sigma^2\right)\\ \hat{\beta} &\sim \mathcal{N}\left(\beta, \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}\right)\end{aligned}$$

Funzione di ripartizione della distribuzione normale standard

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

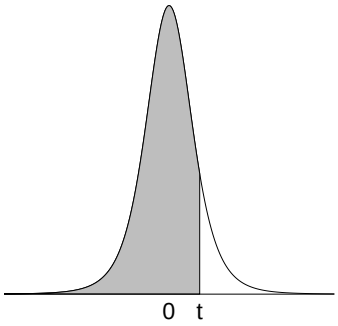


	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997

Tavole costruite con LaTeX, R, Sweave e xtable da Claudio Agostinelli, claudio@unive.it, Ca' Foscari, Venezia, 2005

Alcuni quantili della distribuzione t di Student con r gradi di libertà

$$F_r(t) = \int_{-\infty}^{t} \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\Gamma[r/2](1+x^2/r)^{(r+1)/2}}\, dx$$

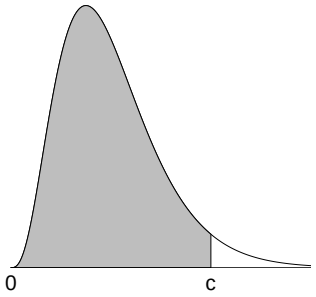


	0.6	0.75	0.9	0.95	0.975	0.99	0.995
1	0.32492	1.00000	3.07768	6.31375	12.70620	31.82052	63.65674
2	0.28868	0.81650	1.88562	2.91999	4.30265	6.96456	9.92484
3	0.27667	0.76489	1.63774	2.35336	3.18245	4.54070	5.84091
4	0.27072	0.74070	1.53321	2.13185	2.77645	3.74695	4.60409
5	0.26718	0.72669	1.47588	2.01505	2.57058	3.36493	4.03214
6	0.26483	0.71756	1.43976	1.94318	2.44691	3.14267	3.70743
7	0.26317	0.71114	1.41492	1.89458	2.36462	2.99795	3.49948
8	0.26192	0.70639	1.39682	1.85955	2.30600	2.89646	3.35539
9	0.26096	0.70272	1.38303	1.83311	2.26216	2.82144	3.24984
10	0.26018	0.69981	1.37218	1.81246	2.22814	2.76377	3.16927
11	0.25956	0.69745	1.36343	1.79588	2.20099	2.71808	3.10581
12	0.25903	0.69548	1.35622	1.78229	2.17881	2.68100	3.05454
13	0.25859	0.69383	1.35017	1.77093	2.16037	2.65031	3.01228
14	0.25821	0.69242	1.34503	1.76131	2.14479	2.62449	2.97684
15	0.25789	0.69120	1.34061	1.75305	2.13145	2.60248	2.94671
16	0.25760	0.69013	1.33676	1.74588	2.11991	2.58349	2.92078
17	0.25735	0.68920	1.33338	1.73961	2.10982	2.56693	2.89823
18	0.25712	0.68836	1.33039	1.73406	2.10092	2.55238	2.87844
19	0.25692	0.68762	1.32773	1.72913	2.09302	2.53948	2.86093
20	0.25674	0.68695	1.32534	1.72472	2.08596	2.52798	2.84534
21	0.25658	0.68635	1.32319	1.72074	2.07961	2.51765	2.83136
22	0.25643	0.68581	1.32124	1.71714	2.07387	2.50832	2.81876
23	0.25630	0.68531	1.31946	1.71387	2.06866	2.49987	2.80734
24	0.25617	0.68485	1.31784	1.71088	2.06390	2.49216	2.79694
25	0.25606	0.68443	1.31635	1.70814	2.05954	2.48511	2.78744
26	0.25595	0.68404	1.31497	1.70562	2.05553	2.47863	2.77871
27	0.25586	0.68368	1.31370	1.70329	2.05183	2.47266	2.77068
28	0.25577	0.68335	1.31253	1.70113	2.04841	2.46714	2.76326
29	0.25568	0.68304	1.31143	1.69913	2.04523	2.46202	2.75639
30	0.25561	0.68276	1.31042	1.69726	2.04227	2.45726	2.75000
31	0.25553	0.68249	1.30946	1.69552	2.03951	2.45282	2.74404
32	0.25546	0.68223	1.30857	1.69389	2.03693	2.44868	2.73848
33	0.25540	0.68200	1.30774	1.69236	2.03452	2.44479	2.73328
34	0.25534	0.68177	1.30695	1.69092	2.03224	2.44115	2.72839
35	0.25528	0.68156	1.30621	1.68957	2.03011	2.43772	2.72381
50	0.25470	0.67943	1.29871	1.67591	2.00856	2.40327	2.67779
75	0.25425	0.67778	1.29294	1.66543	1.99210	2.37710	2.64298
100	0.25402	0.67695	1.29007	1.66023	1.98397	2.36422	2.62589
∞	0.25335	0.67449	1.28155	1.64485	1.95996	2.32635	2.57583

Tavole costruite con LaTeX, R, Sweave e xtable da Claudio Agostinelli, claudio@unive.it, Ca' Foscari, Venezia, 2005

Alcuni quantili della distribuzione χ^2 con r gradi di libertà

$$F_r(c) = \int_{-\infty}^{c} \frac{1}{\Gamma[r/2]2^{r/2}} x^{r/2-1} \exp\left[-\frac{x}{2}\right] \, dx$$



	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995
1	0.0000	0.0002	0.0010	0.0039	0.0158	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.0100	0.0201	0.0506	0.1026	0.2107	4.6052	5.9915	7.3778	9.2103	10.5966
3	0.0717	0.1148	0.2158	0.3518	0.5844	6.2514	7.8147	9.3484	11.3449	12.8382
4	0.2070	0.2971	0.4844	0.7107	1.0636	7.7794	9.4877	11.1433	13.2767	14.8603
5	0.4117	0.5543	0.8312	1.1455	1.6103	9.2364	11.0705	12.8325	15.0863	16.7496
6	0.6757	0.8721	1.2373	1.6354	2.2041	10.6446	12.5916	14.4494	16.8119	18.5476
7	0.9893	1.2390	1.6899	2.1673	2.8331	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.3444	1.6465	2.1797	2.7326	3.4895	13.3616	15.5073	17.5345	20.0902	21.9550
9	1.7349	2.0879	2.7004	3.3251	4.1682	14.6837	16.9190	19.0228	21.6660	23.5894
10	2.1559	2.5582	3.2470	3.9403	4.8652	15.9872	18.3070	20.4832	23.2093	25.1882
11	2.6032	3.0535	3.8157	4.5748	5.5778	17.2750	19.6751	21.9200	24.7250	26.7568
12	3.0738	3.5706	4.4038	5.2260	6.3038	18.5493	21.0261	23.3367	26.2170	28.2995
13	3.5650	4.1069	5.0088	5.8919	7.0415	19.8119	22.3620	24.7356	27.6882	29.8195
14	4.0747	4.6604	5.6287	6.5706	7.7895	21.0641	23.6848	26.1189	29.1412	31.3193
15	4.6009	5.2293	6.2621	7.2609	8.5468	22.3071	24.9958	27.4884	30.5779	32.8013
16	5.1422	5.8122	6.9077	7.9616	9.3122	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.6972	6.4078	7.5642	8.6718	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.2648	7.0149	8.2307	9.3905	10.8649	25.9894	28.8693	31.5264	34.8053	37.1565
19	6.8440	7.6327	8.9065	10.1170	11.6509	27.2036	30.1435	32.8523	36.1909	38.5823
20	7.4338	8.2604	9.5908	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.0337	8.8972	10.2829	11.5913	13.2396	29.6151	32.6706	35.4789	38.9322	41.4011
22	8.6427	9.5425	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7957
23	9.2604	10.1957	11.6886	13.0905	14.8480	32.0069	35.1725	38.0756	41.6384	44.1813
24	9.8862	10.8564	12.4012	13.8484	15.6587	33.1962	36.4150	39.3641	42.9798	45.5585
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9279
26	11.1602	12.1981	13.8439	15.3792	17.2919	35.5632	38.8851	41.9232	45.6417	48.2899
27	11.8076	12.8785	14.5734	16.1514	18.1139	36.7412	40.1133	43.1945	46.9629	49.6449
28	12.4613	13.5647	15.3079	16.9279	18.9392	37.9159	41.3371	44.4608	48.2782	50.9934
29	13.1211	14.2565	16.0471	17.7084	19.7677	39.0875	42.5570	45.7223	49.5879	52.3356
30	13.7867	14.9535	16.7908	18.4927	20.5992	40.2560	43.7730	46.9792	50.8922	53.6720
31	14.4578	15.6555	17.5387	19.2806	21.4336	41.4217	44.9853	48.2319	52.1914	55.0027
32	15.1340	16.3622	18.2908	20.0719	22.2706	42.5847	46.1943	49.4804	53.4858	56.3281
33	15.8153	17.0735	19.0467	20.8665	23.1102	43.7452	47.3999	50.7251	54.7755	57.6484
34	16.5013	17.7891	19.8063	21.6643	23.9523	44.9032	48.6024	51.9660	56.0609	58.9639
35	17.1918	18.5089	20.5694	22.4650	24.7967	46.0588	49.8018	53.2033	57.3421	60.2748
40	20.7065	22.1643	24.4330	26.5093	29.0505	51.8051	55.7585	59.3417	63.6907	66.7660
50	27.9907	29.7067	32.3574	34.7643	37.6886	63.1671	67.5048	71.4202	76.1539	79.4900
60	35.5345	37.4849	40.4817	43.1880	46.4589	74.3970	79.0819	83.2977	88.3794	91.9517
70	43.2752	45.4417	48.7576	51.7393	55.3289	85.5270	90.5312	95.0232	100.4252	104.2149
80	51.1719	53.5401	57.1532	60.3915	64.2778	96.5782	101.8795	106.6286	112.3288	116.3211

Tavole costruite con LaTeX, R, Sweave e xtable da Claudio Agostinelli, claudio@unive.it, Ca' Foscari, Venezia, 2005