$$f(x) = x^{-2} \cdot e^{\frac{1}{x}}$$

1) DOMINIO $x \neq 0$ ; 2) INTERSEZIONE ASSI $f(x) = 0$

CIOE' $x^{-2} \cdot e^{\frac{1}{x}} = 0$ → $x^{-2} = 0$ se $x = 0$ ; → $e^{\frac{1}{x}} = 0$ mai ⇒ IMPOSSIBILE $x \neq 0$

3) POSITIVITA' $x^{-2} \cdot e^{\frac{1}{x}} > 0$ → $x^{-2} > 0$ SEMPRE ; → $e^{\frac{1}{x}} > 0$ SEMPRE ⇒ SEMPRE $f(x) > 0$

4) LIMITI

- $\lim_{x \to \pm\infty} \frac{1}{x^2} \cdot e^{\frac{1}{x}} = \frac{1}{(\pm\infty)^2} \cdot e^{\frac{1}{\pm\infty}} = 0 \cdot e^{0} = 0 \cdot 1 = 0$
- $\lim_{x \to 0^+} \frac{1}{x^2} \cdot e^{\frac{1}{x}} = \frac{1}{0^+} \cdot e^{\frac{1}{0^+}} = +\infty \cdot e^{+\infty} = +\infty \cdot \infty = +\infty$
- $\lim_{x \to 0^-} \frac{1}{x^2} \cdot e^{\frac{1}{x}} = \frac{1}{0^+} \cdot e^{\frac{1}{0^-}} = +\infty \cdot e^{-\infty}$ = FORMA INDETERMINATA

DOVE PREVALE $e^{\frac{1}{x}}$ PERCIO' RISULTA 0.

5) DERIVATA $f'(x) = -2x^{-2-1} \cdot e^{\frac{1}{x}} + x^{-2} \cdot e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)$

$= -2x^{-3} \cdot e^{\frac{1}{x}} - x^{-4} e^{\frac{1}{x}} = e^{\frac{1}{x}} \cdot x^{-3} \left(-2 - x^{-1}\right)$

$f'(x) > 0$ $e^{\frac{1}{x}} > 0$ SEMPRE (I)

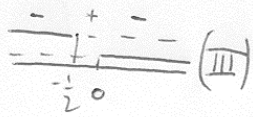
$x^{-3} > 0$ se $x > 0$ (II)

$-2 - \frac{1}{x} > 0$ se $\frac{-2x-1}{x} > 0$

CIOE' se $-2x - 1 > 0 \to -2x > 1$

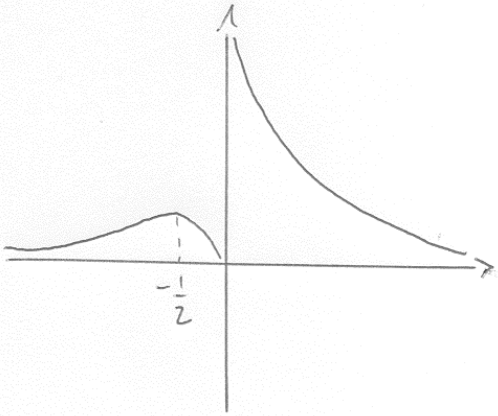
quindi $x < -\frac{1}{2}$

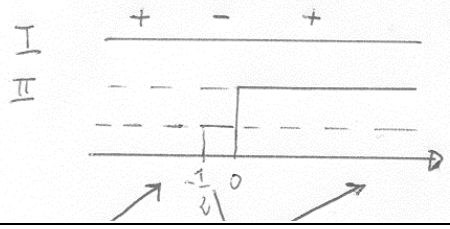
e se $x > 0$




(III)

6) GRAFICO




RIASSUMENDO